(33) $\sum_{n=1}^{\infty} \frac{2^n}{3^n+1}$ Ratio!

as $n \to \infty$ behaves like $\frac{3^n}{3^{n+1}}$

$$\lim_{n\to\infty} \frac{\cancel{2^{n+1}}^{\,2^1}}{3^{n+1}+1} \cdot \frac{3^n+1}{\cancel{2^n}} = \lim_{n\to\infty} 2 \cdot \frac{3^n+1}{3^{n+1}+1}$$

$$= 2 \cdot \frac{1}{3} = \frac{2}{3} < 1 \quad \therefore \text{converges}$$

(37) $\sum_{n=1}^{\infty} \frac{(n+3)!}{3!\, n!\, 3^n}$ Ratio

$\frac{4!}{5!} = \frac{\cancel{4\cdot 3\cdot 2\cdot 1}}{5\cdot \cancel{4\cdot 3\cdot 2\cdot 1}}$

5

$$\lim_{n\to\infty} \frac{\cancel{(n+4)!}^{\,n+4}}{\cancel{3!}\cdot \underbrace{\cancel{(n+1)!}}_{n+1}\cdot \underbrace{\cancel{3^{n+1}}}_{3^1}} \cdot \frac{\cancel{3!}\cdot \cancel{n!}\cdot \cancel{3^n}}{\cancel{(n+3)!}} = \lim_{n\to\infty} \frac{\cancel{n}+4}{(\cancel{n}+1)\cdot 3} = \frac{1}{3} < 1$$

$\therefore$ converges

43 $\sum_{n=1}^{\infty} \frac{n!}{(2n+1)!}$

$$\lim_{n\to\infty} \frac{\cancel{(n+1)!}\ n+1}{\underbrace{\cancel{(2n+3)!}}_{(2n+3)(2n+2)}} \cdot \frac{\cancel{(2n+1)!}}{\cancel{n!}} = \lim_{n\to\infty} \frac{n+1}{(2n+3)(2n+2)}$$

$$= \lim_{n\to\infty} \frac{n+1}{4n^2+10n+6} = 0 < 1 \quad \therefore \text{converges}$$

39 $\dfrac{(-2)^n}{3^n} = \left(\dfrac{-2}{3}\right)^n$

35 $\lim_{n\to\infty} \dfrac{(n+1)^2 \cdot e^{-(n+1)}}{n^2 \cdot e^{-n}}$, where $\dfrac{e^{-(n+1)}}{e^{-n}} = \cancel{e^{-n-1}} = e^{-1}$ and $\dfrac{n+1}{n} \to 1$

$= \dfrac{1}{e} < 1 \quad \therefore$ converges

37 $\displaystyle\sum_{n=1}^{\infty} \frac{(n+3)!}{3!\,n!\,3^n}$ $\underline{\text{Ratio}}$

$$\lim_{n\to\infty} \frac{\cancel{(n+4)!}\ (n+4)}{\cancel{3!}\,\cancel{(n+1)!}\,\cancel{3^{n+1}}} \cdot \frac{\cancel{3!}\,\cancel{n!}\,\cancel{3^n}}{\cancel{(n+3)!}} = \lim_{n\to\infty} \frac{(n+4)}{(n+1)\cdot 3}$$

$(n+1)$ $3'$

$\dfrac{n+4}{n+1} \to 1$

$= \dfrac{1}{3} < 1 \quad \therefore$ series converges

43 $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{(2n+1)!}$

$2(n+1)+1$

$$\lim_{n\to\infty} \frac{\cancel{(n+1)!}\ (n+1)}{\cancel{(2n+3)!}} \cdot \frac{\cancel{(2n+1)!}}{\cancel{n!}} = \lim_{n\to\infty} \frac{\cancel{n+1}}{(2n+3)\,2\,\cancel{(n+1)}}$$

$(2n+3)(2n+2)$

$2(n+1)$

$= \lim_{n\to\infty} \dfrac{1}{2(2n+3)} = 0 < 1 \quad \therefore$ series converges

Even More 9.4 - Radius of Convergence

If a series $\sum |a_n|$ converges, then we say that $\sum a_n$ converges absolutely.

If a series $\sum |a_n|$ converges, then we know that $\sum a_n$ also converges.

"Absolute convergence implies convergence."

Using Ratio Test, if $L<1$ then series converges. Use this fact to find the radius of convergence, R. $\boxed{|x-a|<R}$

Ex1) Find R for $\sum_{n=0}^{\infty} \frac{nx^n}{10^n}$

Use Ratio Test:

$$\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = \lim_{n\to\infty} \left|\frac{(n+1)x^{n+1}}{10^{n+1}}\right| \cdot \left|\frac{10^n}{nx^n}\right|$$

$$\lim_{n\to\infty} \frac{\overset{1}{\cancel{(n+1)}}\ \overset{|x|}{\cancel{|x^{n+1}|}}}{\underset{10^1}{\cancel{10^{n+1}}}} \cdot \frac{\cancel{10^n}}{\underset{1}{\cancel{n}}\ \cancel{|x^n|}} = \lim_{n\to\infty} \frac{|x|}{10} = \frac{|x|}{10}^{L}$$

$$\frac{|x|}{10} < 1$$

$|x| < 10$ Want: $|x-a|<R$

$R=10$

Ex) $\sum_{n=1}^{\infty} \frac{(4x-5)^{2n+1}}{n^{3/2}}$

Use Ratio Test to do $\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|}$.

You get: $|(4x-5)^2| \leftarrow L$

Converge when $|(4x-5)^2| < 1$ $\quad |x-a|<R$

$|4x-5| < 1$

$|x-\frac{5}{4}| < \frac{1}{4}$ $\quad R=\frac{1}{4}$

Ex) Find R for $\sum_{n=0}^{\infty} n!x^n$

Using Ratio Test, you get $\lim_{n\to\infty} (n+1)\cdot|x| = \infty$

When is $(n+1)|x| < 1$? When $x=0$.

Only converges at $x=0$. So $R=0$.

HW: p511 # 7-17 odds
FR